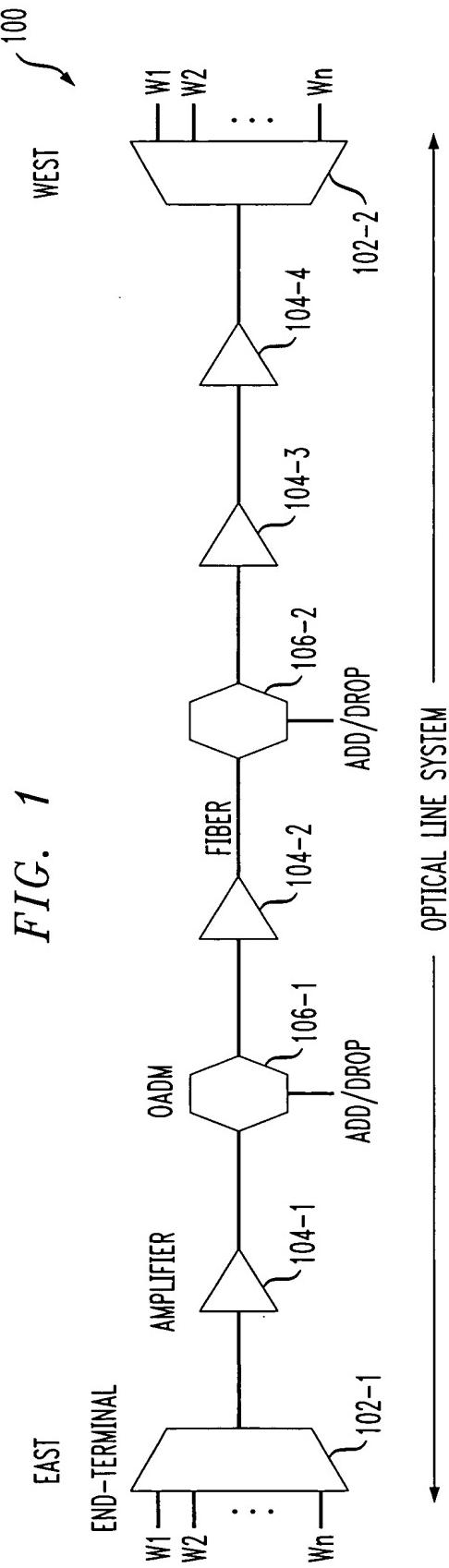




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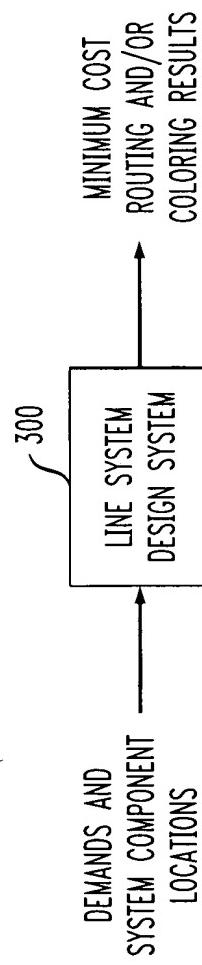
FIG. 2

GENERAL CASE			SPECIAL CASE	
PROBLEM	APPROX LOWER BOUND	APPROX UPPER BOUND	PROBLEM	COMPLEXITY
$(L, D, *)$	$\Omega(\sqrt{s})$	$0 (\sqrt{s})$	$(L, *, E), s = 2$	POLYNOMIAL
$(L, U, NE)$	$1 + 1/s^2$	2	$ C_2  = \infty$	
$(L, U, E)$	NP-HARD	2	$(L, *, NE), s = 2$	POLYNOMIAL
$(C, *, NE)$	IN-APPROXIMABLE		$(L, U, E), s = 2$	4/3-APPROX
$(C, D, E)$	IN-APPROXIMABLE		$(L, D, *), s = 3$	NP-HARD
$(C, U, E)$	NP-HARD	$2(1 + \epsilon)$		

(A)

(B)

FIG. 3





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FIG. 4A

Methodology A

$m(0) = 0$

for  $p = 1$  to line system load/2

{

$l(p) = 0; m(p) = m(p-1) + 2;$

for  $i = 1$  to  $n - 1$

{

$l_i(p) \leftarrow$  load on link  $e_i$

if( $l_i(p) = 0$ )

{

Divide the line system into two line systems;  
one from node 0 to node  $(i-1)$ ; the other from  
node  $i$  to node  $(n-1)$  and call methodology A  
on these line systems separately.

}

if( $l_i(p) > l(p)$ )

{

$l(p) = l_i(p)$

}

}

create a multigraph  $G = (V, E)$ , where  $V = \{0, \dots, n - 1\}$

for all demand  $(i, j)$  in  $D$

{

create an edge  $(i - 1, j)$  in  $G$

}

for  $i = 1$  to  $n - 1$

{

if  $l_i(p) < l(p)$

add an edge  $(i - 1, i)$  in  $G$

}

set the capacity of each edge in  $G$  to 1

find a 2-unit flow from node 0 to node  $(n - 1)$  in  $G$

Let  $p_1$  and  $p_2$  be the path for the flow

For all the demands corresponding to links in  $p_1$ .

{

Assign the color  $c_{m(p)}$  to demand

remove the demand from  $D$

}

For all the demands corresponding to links in  $p_2$

{

Assign the color  $c_{m(p)+1}$  to demand

remove the demand from  $D$

}

{



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## FIG. 4B

Routing Phase:

if  $(L(R_s) \geq n(1 + \epsilon)/\epsilon)$

**Output**  $R_s$

else {

Compute  $D_1 = \{d \in D \mid d \text{ in any routing goes through at least } n/3 \text{ links}\}$

Compute  $D_2 = D - D_1$

Compute  $R_1 = \text{the set of all possible routings for demands in } D_1$

Compute  $R_2 = \text{the set of all possible routings for demands in } D_2$   
in which at most  $3S$  demands are not routed on shortest paths

Compute  $R = R_1 \times R_2$

Compute  $r \in R$  such that  $L(r) = \min_{r' \in R} L(r')$

**Output**  $r$

}

Coloring Phase:

$U = D$

$M = \text{the set of available colors}$

$l = \min_{e_i \in L} l_i(U)$  (the min. load of demands in  $U$ )

while ( $l > 0$ ) {

    Compute  $O = H(U)$  (see below)

    Compute  $m = \{i, j \mid i, j \text{ are the smallest two colors in } M\}$

    Color demands in  $O$  with colors in  $m$

$U = U - O$

$M = M - m$

$l = \min_{e_i \in L} l_i(U)$

}

    if ( $U \neq \emptyset$ ) {

        Color  $U$  using methodology A

“Compute  $O = H(U)$ ”:

Compute  $d_0 = \text{a demand in } U \text{ that goes through the largest number of links in } L$

$O = \{d_0\}$

$L'$  = set of links covered by demands in  $O$

$i = 1$

while ( $L' \neq L$ ) {

    Compute  $D_i = \{d \mid d \in U - O \text{ & } d \text{ overlaps with } d_{i-1}\}$

    Compute  $d_i = \{d \mid d \in D_i \text{ & } d \text{ goes through the largest number of links in } L - L'\}$

$i = i+1$

**output**  $O$

}



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*FIG. 4C*

Methodology B:

$$e_0 = (-1, 0)$$

$$e_{n+1} = (n, n + 1)$$

$$L = L \cup \{e_0, e_{n+1}\}$$

$$D = D \cup \{(0, 0), (n + 1, n + 1)\}$$

for all  $(0 \leq i \leq j \leq n + 1)$  {

$$P(i, j) = \emptyset$$

$$R(i, j) = \emptyset$$

$$\text{best} = 0$$

for all  $(i \leq i' \leq j' \leq j)$  {

$$E_1 = \{e_i, e_{i+1}, \dots, e_{i'}\} \cup \{e_{j'+1}, e_{j'+2}, \dots, e_j\}$$

$$E_2 = e_{i'+1}, e_{i'+2}, \dots, e_{j'}$$

Compute coloring  $C$  using methodology b1 where  $E_1$  ( $E_2$ ) links are colored with 1 (2) steps

if( $C \neq \emptyset$ ) {

    if( $i' - i + j - j' + 1 \geq \text{best}$ ) {

$$R(i, j) = C$$

$$\text{best} = i' - i + j - j' + 1$$

}

}

}

$$\text{Compute } L_1 = \{e_i \mid e_i \in L, l_i \leq |C_1|\}$$

for all  $(e_i, e_j \in L_1)$  {

$$\text{Compute } D_{i,j} = \{d \mid d \in D, d \text{ goes through either link } e_i, e_j\}$$

    Compute  $P_{i,j} = \text{coloring obtained by coloring the interval graph } D_{i,j} \text{ with colors in } C_1$

}

for all  $(e_i, e_j \in L_1, i < j)$  {

$$\text{best} = 0$$

    for all  $(m, i < m < j)$  {

        Compute the coloring  $K = P(i, m) + P(m, j)$

        If( $K = \emptyset$ ) continue

        Compute  $n = \text{number of links that are in one step in } K$

        if( $\text{best} < n$ ) {

$$\text{best} = n$$

$$C = K$$

}

}

    Compute  $n = \text{number of links that are in one step in } R(i, j)$

    if( $\text{best} < n$ ) {

$$\text{best} = n$$

$$C = R(i, j)$$

}

$$P(i, j) = C$$

}

**Output**  $P(0, n + 1)$



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## FIG. 4D

### Methodology b1:

Compute  $C$  = interval graph coloring of demands  $D_1$  using colors in  $C_1$   
if( $C == \emptyset$ ) **Output**  $C$ .

Compute  $C'$  = interval graph coloring of the demands in  $D - D_1$  using first available colors

**Output**  $C' \cup C$

## FIG. 4E

### Methodology c1:

$V = \{0, 1, \dots, n-1\}$

$E = \emptyset$

for all demands  $((i, j) \in D - D_1)$  {

$E = E \cup \{(i - 1, j)\}$

Directed link  $(i - 1, j)$  has unit capacity

}

for all links  $(e_i \in L)$  {

$E = E \cup \{(i - 1, i)\}$

Directed link  $(i - 1, i)$  has capacity  $|C_1| + |C_2| - l_i$

}

Graph  $G = (V, E)$

Compute maxFlow = Max. Flow  $f$  in  $G$  from node 0 to node  $n - 1$

if(maxFlow <  $|C_2|$ ) **Output**  $\emptyset$

Compute  $F_1 = \{d | f$  puts zero flow on the edge  $(i - 1, j)$  where demand  $d = (i, j)\}$

Compute  $F_1 = F_1 \cup D_1$

Compute  $K_1$  = coloring that colors demands in  $F_1$  with colors in  $C_1$  only using interval graph coloring

Compute  $K_2$  = coloring that colors demands in  $D - F_1$  with colors in  $C_2$  only using interval graph coloring

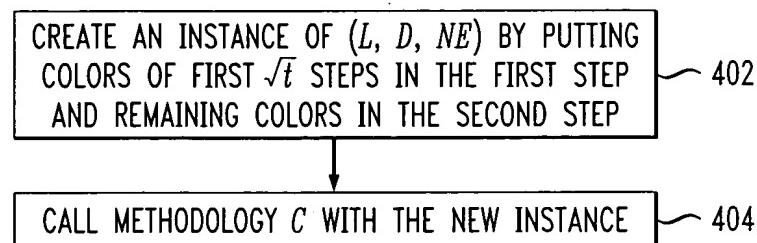
**Output**  $K = K_1 \cup K_2$



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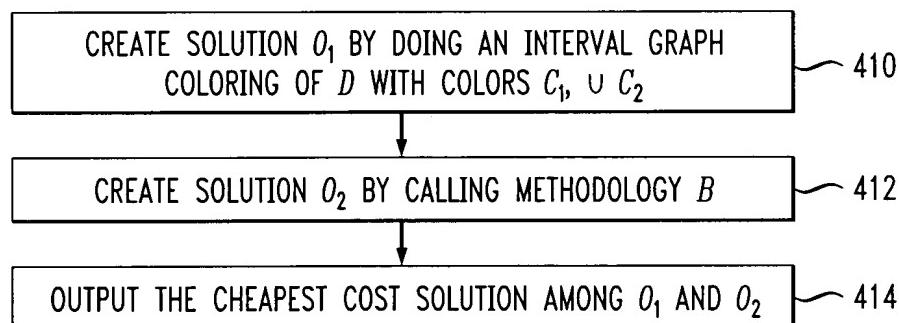
*FIG. 4F*

METHODOLOGY D:



*FIG. 4G*

METHODOLOGY E:



*FIG. 5*

